International Journal of Engineering, Science and Mathematics
Vol. 9 Issue 5, May 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

M^X/G/1 QUEUEING SYSTEM WITH SERVER BREAKDOWN, BERNOULLI VACATION SCHEDULE UNDER RESTRICTED ADMISSIBILITY POLICY

DR. DEEPA CHAUHAN

ABSTRACT

In this paper, we analyze a model which deals with the aspects concerning the control of the arrival process with second optional repair and Bernoulli vacation schedule. The paper deals with M^x/G/1 queueing system where after completion of a service the server either goes for a vacation of random length with probability $\theta (0 \le \theta \le 1)$ or may continue to serve the next customer with probability $(1-\theta)$, if any. Both service time and vacation time follow general distribution. Unlike the usual batch arrivals queueing model, there is restriction over the admissibility of batch arrivals in which not all the arriving batches are allowed to join the queue at all times. The restricted admissibility policy differs during a busy period and a vacation period. We obtain the time dependent probability generating functions in terms of their Laplace transforms and corresponding steady state results explicitly.

KEYWORDS:

M^X/G/1 queue, Server Breakdown, Controlled admissibility policy, Bernoulli vacation schedule.

Author correspondence:

Deepa Chauhan,
Deptt. Of Applied Science & Humanities,
Axis Institute of Technology and Management,
Kanpur

1. INTRODUCTION

Control of queues is one of the most significant and interesting area of queueing theory. One of the queue control is, control on admission of arriving customers. In these problems, either the arrival rate can be modified or the customer can be refused admission. In some of queueing models, queue length is controlled by the rejection of some of incoming arriving customers, in some models, the customers themselves control the decision to enter in to the system. Such a control policy is called restricted admissibility policy and which has been studied by many authors. Ghosh and Weerasinghe (2010) addressed a rate control problem associated with a single server Markovian queueing system with customer abandonment in heavy traffic. Transient solution of an $M^X/G/1$ queue queueing model under restricted admissibility policy has been analyzed by Ayyappan and Shyamala (2013).

Presently, most of the studies are devoted to batch arrival vacation models under different vacation policies because of its interdisciplinary characteristic. Steady state behavior of an $M^X/G/1$ queue with general retrial time and Bernoulli vacation schedule for an unreliable server with delayed repair has been studied by Choudhury and Ke (2012) . Gao and Liu (2013) obtained stationary performance measures for M/G/1 queue with vacation interruption under Bernoulli vacation schedule.

In the present paper, we consider unreliable $M^x/G/1$ queue under controlled admissibility policy with the concept of Bernoulli vacation schedule. The paper is organized as follows. In section 2, we define the underlying assumptions and notations of the system under study. The analysis based on supplementary variables and generating function approach, is given in section 3. Last section is devoted to concluding remarks.

2. MODEL DESCRIPTION

The following assumptions are made to describe the model mathematically:

- Customers arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda a_i \Delta t(i = 1, 2, 3, ...)$ be first order probability that a batch of $'i'$ customers arrives at the system during a short interval of the time $(t, t+\Delta t)$, where $0 \le a_i \le 0$, $\sum_{i=1}^{\infty} a_i = 1$ and $\lambda > 0$ is the mean arrival rate of the batches.
- Each customer undergoes service provided by a single server on FCFS basis. The service time follows general probability distribution with distribution function (DF) $B(x)$, density function $b(x)$ Laplace Stieltjes transform (LST) $B^*(s)$ and finite moments $E(B^k)\,(k \ge 1)$.
- Let $\mu(x)\,dx$ be the conditional probability of completion of the service during the interval $(x, x+dx]$ given that elapsed time is x , so that $\mu(x) = \dfrac{b(x)}{1 - B(x)}$ and therefore , $b(v) = \mu(v)\, e^{-\int_0^v \mu(x)\,dx}$
- Further it is assumed that not all batches are allowed to join the system at all times. Let ξ_1 $(0 \le \xi_1 \le 1)$ be the probability that an arriving batch will be allowed to join the system when server is not on vacation and ξ_2 $(0 \le \xi_2 \le 1)$ be the probability that an arriving batch will be allowed to join the system during the vacation period.
- As soon as the service of a customer is completed the server may go for a vacation of random length V with probability $\theta\,(0 \le \theta \le 1)$ or may continue to serve the next customer, if any, with probability $(1-\theta)$. Assuming that the vacation random variable $V(s)$ follows a general probability distribution with Distribution function $v(s)$, LST $V^*(s)$ and finite moment $E(V^k)\,(k \ge 1)$ and is independent of service time random variable and the arrival process.

- Let $\vartheta(x)\,dx$ be the conditional probability of completion of a vacation during the interval $(x, x+dx]$ given that elapsed time is x, so that $\vartheta(x) = \dfrac{v(x)}{1-V(x)}$ and therefore, $v(s) = \vartheta(s)\, e^{-\int_0^s \vartheta(x)\,dx}$

Chapman Kolmogorov equations governing the model are constructed as follows:

$$\frac{\partial}{\partial t} P_n(x,t) + \frac{\partial}{\partial x} P_n(x,t) + (\lambda + \mu(x)) P_n(x,t) = \lambda(1-\xi_1) P_n(x,t) + \lambda\,\xi_1 \sum_{i=1}^{n-1} a_i\, P_{n-i}(x,t);\ n \ge 1 \tag{1}$$

$$\frac{\partial}{\partial t} P_0(x,t) + \frac{\partial}{\partial x} P_0(x,t) + (\lambda + \mu(x)) P_0(x,t) = \lambda(1-\xi_1) P_0(x,t) \tag{2}$$

$$\frac{\partial}{\partial t} V_n(x,t) + \frac{\partial}{\partial x} V_n(x,t) + (\lambda + \vartheta(x)) V_n(x,t) = \lambda(1-\xi_2) V_n(x,t) + \lambda\,\xi_2 \sum_{i=1}^{n-1} a_i\, V_{n-i}(x,t);\ n \ge 1 \tag{3}$$

$$\frac{\partial}{\partial t} V_0(x,t) + \frac{\partial}{\partial x} V_0(x,t) + (\lambda + \vartheta(x)) V_0(x,t) = \lambda(1-\xi_2) V_0(x,t) \tag{4}$$

$$\frac{d}{dt} Q(t) + \lambda\,\xi_1\, Q(t) = \lambda\xi_1 (1-\xi_2) Q(t) + \xi_2 (1-\theta) \int_0^\infty P_0(x,t)\,\mu(x)\,dx + \xi_1 \int_0^\infty V_0(x,t)\,\vartheta(x)\,dx \tag{5}$$

The above equations are solved to be solved with the following boundary conditions at $x = 0$

$$P_0(0,t) = a_1\, \lambda\, \xi_1 \xi_2\, Q(t) + \xi_2 (1-\theta) \int_0^\infty P_1(x,t)\,\mu(x)\,dx + \xi_1 \int_0^\infty V_1(x,t)\, v(x)\,dx \tag{6}$$

$$P_n(0,t) = a_{n+1}\, \lambda\, \xi_1 \xi_2\, Q(t) + \xi_2 (1-\theta) \int_0^\infty P_{n+1}(x,t)\,\mu(x)\,dx + \xi_1 \int_0^\infty V_{n+1}(x,t)\,\vartheta(x)\,dx;\ n \ge 0 \tag{7}$$

$$\xi_1\, V_n(0,t) = \xi_2\, \theta \int_0^\infty P_n(x,t)\,\mu(x)\,dx;\ n \ge 0 \tag{8}$$

Next we define the following probability Generating Functions:

$$P(x,z,t) = \sum_{n=0}^\infty z^n\, P_n(x,t);\ P(z,t) = \sum_{n=0}^\infty z^n\, P_n(t);\ V(x,z,t) = \sum_{n=0}^\infty z^n\, V_n(x,t);\ V(z,t) = \sum_{n=0}^\infty z^n\, V_n(t)$$

$$a(z) = \sum_{n=1}^\infty a_n\, z^n$$

3. THE ANALYSIS

Theorem 1: The marginal generating functions when the server is busy, on vacations, under repair with first essential repair and optional second repair, respectively, are

$$\overline{P}(z,s) = \overline{P}(0,z,s) \left\{ \frac{1 - \overline{B}(s + \lambda\xi_1 (1-a(z)) + \alpha)}{s + \lambda\xi_1 (1-a(z)) + \alpha} \right\}$$

$$\overline{V}(z,s) = \left(\frac{\xi_2}{\xi_1}\right) \theta\, P(0,z,s)\, B(s + \lambda\xi_2 (1-a(z)) + \alpha) \left\{ \frac{1 - \overline{V}(s + \lambda\xi_2 (1-a(z)))}{s + \lambda\xi_2 (1-a(z))} \right\}$$

Where

$$\overline{P}(0,z,s)=\frac{\lambda\,\xi_1\xi_2\,(a(z)-1)\,\overline{Q}(s)+\left(1-s\,\overline{Q}(s)\right)}{\eta}$$

Proof:

Taking Laplace transforms of equations (1)-(8)

$$\frac{\partial}{\partial x}\overline{P}_n(x,s)+\left(s+\lambda+\mu(x)+\alpha\right)\overline{P}_n(x,s)=\lambda(1-\xi_1)\overline{P}_n(x,s)+\lambda\,\xi_1\sum_{i=1}^{n-1}a_i\,\overline{P}_{n-i}(x,s);\ n\geq 1 \quad (9)$$

$$\frac{\partial}{\partial x}\overline{P}_0(x,s)+\left(s+\lambda+\mu(x)+\alpha\right)\overline{P}_0(x,s)=\lambda(1-\xi_1)\overline{P}_0(x,s) \quad (10)$$

$$\frac{\partial}{\partial x}\overline{V}_n(x,s)+\left(s+\lambda+\vartheta(x)\right)\overline{V}_n(x,s)=\lambda(1-\xi_2)\overline{V}_n(x,s)+\lambda\,\xi_2\sum_{i=1}^{n-1}a_i\,\overline{V}_{n-i}(x,s);\ n\geq 1 \quad (11)$$

$$\frac{\partial}{\partial x}\overline{V}_0(x,s)+\left(s+\lambda+\vartheta(x)\right)\overline{V}_0(x,s)=\lambda(1-\xi_2)\overline{V}_0(x,s) \quad (12)$$

$$(s+\lambda)\xi_1\,\overline{Q}(s)=\lambda\xi_1\,(1-\xi_2)\overline{Q}(s)+\xi_2(1-\theta)\int_0^\infty\overline{P}_0(x,s)\,\mu(x)\,dx+\xi_1\int_0^\infty\overline{V}_0(x,s)\,\vartheta(x)\,dx \quad (13)$$

Boundary conditions:

$$\xi_2\overline{P}_0(0,s)=a_1\,\lambda\,\xi_1\xi_2\,\overline{Q}(s)+\xi_2(1-\theta)\int_0^\infty\overline{P}_1(x,s)\,\mu(x)\,dx+\xi_1\int_0^\infty\overline{V}_1(x,s)\,\vartheta(x)\,dx \quad (14)$$

$$\xi_2\overline{P}_n(0,s)=a_{n+1}\,\lambda\,\xi_1\xi_2\,\overline{Q}(s)+\xi_2(1-\theta)\int_0^\infty\overline{P}_{n+1}(x,s)\,\mu(x)\,dx+\xi_1\int_0^\infty\overline{V}_{n+1}(x,s)\,\vartheta(x)\,dx;\ n\geq 0 \quad (15)$$

$$\xi_1\,\overline{V}_n(0,s)=\xi_2\,\theta\int_0^\infty\overline{P}_n(x,s)\,\mu(x)\,dx;\ n\geq 0 \quad (16)$$

Now multiply equation (9) and (10) by z^n respectively and summing over all possible $'n'$ and using probability generating functions, we obtain

$$\frac{\partial}{\partial x}\overline{P}(x,z,s)+\left(s+\lambda\xi_1(1-a(z))+\mu(x)+\alpha\right)\overline{P}(x,z,s)=0 \quad (17)$$

Performing similar operation on equations (11) to (13)

$$\frac{\partial}{\partial x}\overline{V}(x,z,s)+\left(s+\lambda\xi_2(1-a(z))+\vartheta(x)\right)\overline{V}(x,z,s)=0 \quad (18)$$

For boundary conditions, multiply equation (15) by z and (16) by z^{n+1} respectively and summing over all possible $'n'$ and using probability generating functions, we obtain

$$z\xi_2\overline{P}(0,z,s)=\lambda\,\xi_1\xi_2\,(a(z)-1)\,\overline{Q}(s)+\left(1-s\,\overline{Q}(s)\right)+\xi_2(1-\theta)\int_0^\infty\overline{P}(x,z,s)\,\mu(x)\,dx+\xi_1\int_0^\infty\overline{V}(x,z,s)\,\vartheta(x)\,dx;\ n\geq 0 \quad (19)$$

Similarly

$$\xi_1\,\overline{V}(0,z,s)=\xi_2\,\theta\int_0^\infty\overline{P}(x,z,s)\,\mu(x)\,dx;\ n\geq 0 \quad (20)$$

Integrating eq. (17), we get

$$\overline{P}(x,z,s)=\overline{P}(0,z,s)\exp\left\{-\left(s+\lambda\xi_1(1-a(z))+\alpha\right)x-\int_0^x\mu(t)\,dt\right\} \quad (21)$$

Where $\overline{P}(0,z,s)$ is given by eq. (19)

Integrate (21) by parts w.r.t. x,

$$\overline{P}(z,s)=\overline{P}\,(0,z,s)\left\{\frac{1-\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right)}{s+\lambda\xi_1\,(1-a(z))}\right\} \tag{22}$$

Where,

$$\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right)=\int_0^\infty e^{-(s+\lambda\xi_1\,(1-a(z)))x}\,dB(x) \tag{23}$$

is the Laplace- Stieltjes transform of the service time $B(x)$

Multiply (21) by $\mu(x)$ and integrating over x, we get

$$\int_0^\infty \overline{P}(x,z,s)\,\mu(x)\,dx=\overline{P}\,(0,z,s)\,\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right) \tag{24}$$

From eqs. (20) & (24)

$$\xi_1\,\overline{V}(0,z,s)=\xi_2\,\theta\,\overline{P}\,(0,z,s)\,\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right);\ n\geq 0 \tag{25}$$

Integrating (16) , we get

$$\overline{V}(x,z,s)=\overline{V}\,(0,z,s)\exp\left\{-\left(s+\lambda\xi_2\,(1-a(z))\right)x-\int_0^x \vartheta(t)\,dt\right\} \tag{26}$$

From eq. (25) and (26)

$$\overline{V}(x,z,s)=\left(\frac{\xi_2}{\xi_1}\right)\theta\,\overline{P}\,(0,z,s)\,\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right)\exp\left\{-\left(s+\lambda\xi_2\,(1-a(z))\right)x-\int_0^x \vartheta(t)\,dt\right\} \tag{27}$$

Again integrating eq. (27) by parts w.r.t x :

From (35)

$$\overline{V}(z,s)=\left(\frac{\xi_2}{\xi_1}\right)\theta\,P(0,z,s)\,B\left(s+\lambda\xi_2\,(1-a(z))\right)\left\{\frac{1-\overline{V}\left(s+\lambda\xi_2\,(1-a(z))\right)}{s+\lambda\xi_2\,(1-a(z))}\right\} \tag{28}$$

Where,

$$\overline{V}\left(s+\lambda\xi_1\,(1-a(z))\right)=\int_0^\infty e^{-(s+\lambda\xi_2\,(1-a(z)))x}\,dV(x) \tag{29}$$

is the Laplace- Stieltjes transform of the vacation time $V(x)$. Now multiply (37) by $\vartheta(x)$ and integrating over x, we get

$$\int_0^\infty \overline{V}(x,z,s)\,\vartheta(x)=\left(\frac{\xi_2}{\xi_1}\right)\theta\,\overline{P}(0,z,s)\,\overline{B}\left(s+\lambda\xi_2\,(1-a(z))\right)\overline{V}\left(s+\lambda\xi_2\,(1-a(z))\right) \tag{30}$$

Using eq. (24), eq. (17) becomes

$$z\xi_2\overline{P}(0,z,\ s)=$$

$$\lambda\,\xi_1\xi_2\,(a(z)-1)\,\overline{Q}(s)+\left(1-s\,\overline{Q}(s)\right)+(1-\psi)\xi_2\,\beta_1\,\alpha\,z\overline{P}\,(0,z,s)\left\{\frac{1-\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right)}{\left(s+\lambda\xi_1\,(1-a(z))\right)\left(s+\lambda\xi_1(1-a(z))+\beta_1\right)}\right\}$$

$$+\xi_2\,\beta_2\,\psi\,\beta_1\,\alpha\,z\overline{P}\,(0,z,s)\left\{\frac{1-\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right)}{\left(s+\lambda\xi_1\,(1-a(z))+\alpha\right)\left(s+\lambda\xi_1(1-a(z))+\beta_1\right)\left(s+\lambda\xi_1\,(1-a(z))+\beta_2\right)}\right\}$$

$$+\xi_2(1-\theta)\overline{P}\,(0,z,s)\,\overline{B}\left(s+\lambda\xi_1\,(1-a(z))\right)$$

$$+\xi_2\,\theta\,\overline{P}(0,z,s)\,\overline{B}\left(s+\lambda\xi_2\,(1-a(z))+\alpha\right)\overline{V}\left(s+\lambda\xi_2\,(1-a(z))\right)$$

$$\Rightarrow \overline{P}\,(0,z,s)=\frac{\lambda\,\xi_1\xi_2\,(a(z)-1)\,\overline{Q}(s)+\left(1-s\,\overline{Q}(s)\right)}{\eta}$$

Theorem 2: The steady state probabilities when the server is busy while rendering service, on vacations, under repair with first essential repair and optional second repair, respectively, are given as follow

$$P(z)=\left\{\frac{1-\bar{B}(s+\lambda\xi_1(1-a(z))+\alpha)}{s+\lambda\xi_1(1-a(z))+\alpha}\right\}\left\{\frac{\lambda\,\xi_1\xi_2\,(a(z)-1)\,\bar{Q}}{\eta}\right\} \tag{31}$$

$$V(z)=\left(\frac{\xi_2}{\xi_1}\right)\theta\;B(s+\lambda\xi_2(1-a(z))+\alpha)\left\{\frac{1-\bar{V}(s+\lambda\xi_2(1-a(z)))}{s+\lambda\xi_2(1-a(z))}\right\}\left\{\frac{\lambda\,\xi_1\xi_2\,(a(z)-1)\,\bar{Q}}{\eta}\right\} \tag{32}$$

Proof: By Using Tauberian property $Lt_{s\to 0}s\bar{f}(s)=Lt_{t\to\infty}f(t)$ in eqs. (32) (38), (41), (42) and (43), we get steady state probabilities when the server is busy, on vacations, respectively.

Theorem 3: The probability generating function of the number of the customers in the queue is given by

$$W(z)=\left(\frac{1-\bar{B}(s+\lambda\xi_1(1-a(z))+\alpha)}{s+\lambda\xi_1(1-a(z))+\alpha}\right)+\left(\frac{\xi_2}{\xi_1}\right)\theta\;B(s+\lambda\xi_2(1-a(z))+\alpha)\left(\frac{1-\bar{V}(s+\lambda\xi_2(1-a(z)))}{s+\lambda\xi_2(1-a(z))}\right)$$
$$+\left\{\frac{\lambda\,\xi_1\xi_2\,(a(z)-1)\,\bar{Q}}{\eta}\right\}$$

Proof: Probability generating function of the number of the customers in the queue $W(z)$ irrespective to the state of system is given

$$W(z)=P(z)+V(z)$$

$$W(z)=\left(\frac{1-\bar{B}(s+\lambda\xi_1(1-a(z))+\alpha)}{s+\lambda\xi_1(1-a(z))+\alpha}\right)+\left(\frac{\xi_2}{\xi_1}\right)\theta\;B(s+\lambda\xi_2(1-a(z)))\left(\frac{1-\bar{V}(s+\lambda\xi_2(1-a(z)))}{s+\lambda\xi_2(1-a(z))}\right)$$
$$+\left\{\frac{\lambda\,\xi_1\xi_2\,(a(z)-1)\,\bar{Q}}{\eta}\right\}$$

4. CONCLUDING REMARKS

The research on queueing theory has been extensively developed due to a lot of significance in the problems relating with decision making process and it has made a tremendous impact in industry and logistic sector from its significant applications in many other areas like population studies, health sectors, manufacturing and production sections etc..In the present paper, we have considered $M^x/G/1$ queue with batch arrival and modified Bernoulli schedule server vacations. In many congestion situations just before a service starts, the server has the option to control the queue by controlling the admissions of arriving batches. Such a model may find applications in many day to day real life queueing situations. Further our model assumes that the server vacations are based on Bernoulli schedule which means that just after completing a service selected by the customer, the server may take vacation of random length or may continue staying in the system. The concepts of Bernoulli schedule vacation, batch arrival and controlled permissibility have been incorporated together in our queueing model which has potential applicability in manufacturing, computer and communication systems, etc..

REFERENCES

1. Ayyappan, G. and Shyamala, S. (2013). Transient solution of an $M^x/G/1$ queueing model with Bernoulli feedback, random breakdowns, Bernoulli scheduled server vacation and restricted admissibility policy.Asian Journal of Matehmatics and Applications. Vol. 23: 1-11.

2. Choudhury, G. and Ke, J.C. (2012). A batch arrival retrial queue with general retrial times under Bernoulli vacation schedule for unreliable server and delayed repair. Applied Mathematical Modeling. Vol. 36 : 255-269.

3. Gao, S. and Liu, Z. (2013). An M/G/1 queue with single working vacation and vacation interruption under Bernoulli vacation schedule. Applied Mathematical Modeling. Vol. 37: 1564-1579.

4. Ghos, A.P. and Weerasinghe, A.P. (2010). Optimal buffer size and dynamic rate control for a queueing system with impatient customers in heavy traffic. Stochastic Process and their Applications. Vol. 120: 2103-2114.

References

The main references are international journals and proceedings. All references should be to the most pertinent and up-to-date sources. References are written in APA style of Roman scripts. Please use a consistent format for references – see examples below (9 pt):

[1] Arya, K.S., "Analysis and Simplification of Three-Dimensional Space Vector PWM for Three-Phase Four-Leg Inverters," *IEEE Transactions on Industrial Electronics,* vol. 58, pp. 450-464, Feb 2011.

[2] Berto, M.C. and Vruce K., "Implementation of a Fuzzy PI Controller for Speed Control of Induction Motors Using FPGA," *Journal of Power Electronics,* vol. 10, pp. 65-71, 2010.

[3] Newmark, P. "Common Mode Circulating Current Control of Interleaved Three-Phase Two-Level Voltage-Source Converters with Discontinuous Space-Vector Modulation," *2009 IEEE Energy Conversion Congress and Exposition, Vols 1-6,* pp. 3906-3912, 2009.

[4] Zed, Yinhai. "A Novel SVPWM Modulation Scheme," in *Applied Power Electronics Conference and Exposition, 2009. APEC 2009. Twenty-Fourth Annual IEEE*, 2009, pp. 128-131.